

## Algorithm to optimize a fuzzy model of students academic achievement in higher education

**Silvia Ramos Cabral**

Universidad de Guadalajara

[silviarc@cutonala.udg.mx](mailto:silviarc@cutonala.udg.mx)

**Juan Carlos González Castolo**

Universidad de Guadalajara

[jcgcastolo@cucea.udg.mx](mailto:jcgcastolo@cucea.udg.mx)

**Sara Catalina Hernández Gallardo**

Universidad de Guadalajara

[shernand@cencar.udg.mx](mailto:shernand@cencar.udg.mx)

### Abstract

This paper shows an algorithm called *Fuzzy Sets Optimized with a Genetic Algorithm* (FSOGA) in order to obtain an *Optimized Fuzzy Model of Academic Achievement* (OFMAA). Student academic information of higher education as the aptitude test and the grade point average of the first semester is used. The OFMAA is based in Fuzzy Logic's Theory because the *Academic Achievement* (AA) is a complex phenomenon that is more natural to describe it with a human reasoning's approximation. The OFMAA is a set of *Fuzzy Sets* (FSs) and if-then rules. In order to find the best FSs that describe the AA, it is used a *Genetic Algorithm* (GA) which contains restrictions that print particular and desirable characteristics in FSs.

**Keywords:** Academic Achievement Model, Optimized Model, Model with Fuzzy Logic Sets.

---

## Introduction

Techniques of *Artificial Intelligence* (AI) as *Artificial Neural Networks* (ANNs) (Warren, 1994), *Fuzzy Logic Systems* (Mendel, 1995), *GAs* (Tank, Man, Kwongs, & He, 1996), are gaining great popularity in recent years, because of their effectiveness in solving complex problems in areas of control (Zhi & Han-Xion, 2005), engineering (Rowcliffe & Feng, 2008), optimization (Render & Flasse, 1996), etc. GA's theory proposes the use of a solutions population and they are refined with the idea that the ancestors pass the best characteristics to their descendants from generation to generation. Therefore is acceptable to think that the GA is robust, efficient and effective to solve some complex problems and achieve optimized solutions.

On the other hand, *Fuzzy Logic* (FL) has the facility to process data, according to linguistic variables and degrees of memberships, so FL is able to model imprecise reasoning and complex systems (Wang & Langari, 1996). Special problems are resolved with hybrid theory like GA and FL (Varnamkhasti, Abu Bakar & Leong, 2012).

In this paper we consider the problem of AA because it is known that the administrative policy of any higher education institution requires improve the terminal efficiency that is AA's consequence. The AA model helps to identify and attend opportunely some problems that are affecting terminal efficiency. When the problems are identified the activation of institutional support mechanisms that minimize the student failure can be facilitated. The model is defined as a representation of an object, system, or even an idea (Shannon & Bernal, 2008). In this sense, models are used for communication, entertainment, instruction, prediction and experimentation (Tarifa, 2001). In education there are some models that are used to classify learning styles (Durán & Costaguta, 2007) and to identify the weaknesses of students in early stages of study (Lye, Ng, Hassan, Goh, Law & Ismail, 2010) Other approaches focus on using student models to streamline the computer systems of student aid known as intelligent tutors (Larrañaga, Elorriaga, Rueda & Urruarte, 2004).

The AA problem is multifactorial however it is mapped only to student scores in order to describe the FSOGA proposed. OFMAA is specified as a series of rules and fuzzy sets. This paper is organized as follow: Section II is an overview of FS and GAs theory. Techniques related to use of GAs to find fuzzy sets and metric used to make AA estimation and associated concepts, there are also included. FSOGA algorithm is explained in Section III. OFMAA evaluation and analysis are presented in Section IV. Finally, the conclusions are show in Section V.

## Background

### A. Possibility Theory

In possibility theory, a FS  $\tilde{a}$  is used to delimit ill-known values or to represent values characterized by linguistic variable expressions. The FS  $\tilde{a}$  in  $\mathbf{X}$  is characterized by a membership function  $\mu_{\tilde{a}}(x)$  which associates to each point  $x$  in  $\mathbf{X}$  a real number in the interval  $[0, 1]$ ; the value  $\mu_{\tilde{a}}(x)$  represents the “degree of membership” of  $x$  in  $\tilde{a}$  (Zadeh, 1965).

A FS can be defined in a triangular form; thus a fuzzy set  $\tilde{a}$  can be characterized as  $\tilde{a} = (a_1, a_2, a_3)$  such that  $a_1, a_2, a_3 \in \mathbb{R}^+$ , where  $(a_2)$  and  $(a_1, a_3)$  are the core and support of  $\tilde{a}$  respectively.  $\tilde{a} \uparrow = (a_1, a_2)$  denotes a subset of  $\tilde{a}$  where the values  $\mu_{\tilde{a}}(x)$  grow towards 1; similarly  $\tilde{a} \downarrow = (a_2, a_3) \subseteq \tilde{a}$  denotes the decreasing values of  $\mu_{\tilde{a}}(x)$ . Figure 1(a) illustrates these notions.

*Definition 1:* The intersection and union of FS are defined in terms of *min* and *max*.  $(\tilde{a} \cap \tilde{b}) = \min(\tilde{a}, \tilde{b}) = \min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(x))$  such that  $x \in \text{support of } \tilde{a} \wedge \tilde{b}$  and  $(\tilde{a} \cup \tilde{b}) = \max(\tilde{a}, \tilde{b}) \equiv \max(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(x))$  such that  $x$  belongs to the support of  $\tilde{a} \vee \tilde{b}$ , where the *min*(*max*) operator gets the minimum (maximum)  $x$  of  $\mathbf{X}$ , Fig. 1(c)(d). We use these intersection and union operators as a *t*-norm and a *s*-norm, respectively. The null element for *min* (*max*) operation is 1(0), (González-Castolo & López-Mellado, 2007).

Definition 2: An  $\alpha$  – cut of the membership function  $\mu_{\tilde{a}}(x)$  (denoted  $\tilde{a}_{\alpha}$ ) is the set of all  $x$  such that  $\mu_{\tilde{a}}(x)$  is greater than or equal to  $\alpha$  where  $\alpha \in [0, 1]$ . Similarly, a strong  $\alpha$  – cut (denoted  $\tilde{a}_{\alpha+}$ ) is the set of all  $x$  such that  $\mu_{\tilde{a}}(x)$  is strictly greater than  $\alpha$ , (Klir & Yuan, 1995).

$$\tilde{a}_{\alpha} = \{x \in X \mid \mu_{\tilde{a}}(x) \geq \alpha, \alpha \in [0, 1]\} \tag{1}$$

$$\tilde{a}_{\alpha+} = \{x \in X \mid \mu_{\tilde{a}}(x) > \alpha, \alpha \in [0, 1]\} \tag{2}$$

Definition 3: The set of all levels  $\alpha$  in  $[0, 1]$  that represent distinct  $\alpha$ – cut of a given fuzzy set  $\tilde{a}$  is called a level set of  $\tilde{a}$  and it denoted as  ${}_{\alpha}\Lambda_{\tilde{a}}$  (Klir & Yuan, 1995).

$${}_{\alpha}\Lambda_{\tilde{a}} = \{\alpha \mid \mu_{\tilde{a}}(x) = \alpha \text{ for some } x \in X\} \tag{3}$$

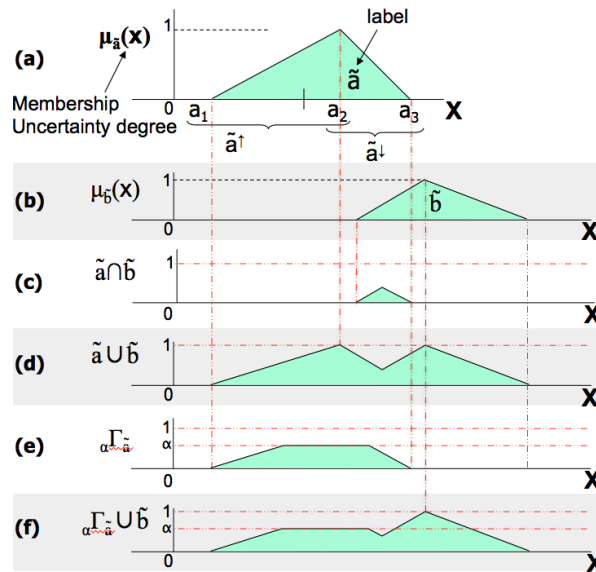


Fig. 1. Fuzzy sets and operations.

Definition 4: A FS  $\tilde{a}$  without  $\tilde{a}_{\alpha+}$  (denoted  ${}_{\alpha}\Gamma_{\tilde{a}}$ ) is the set of all  $x$  such that  $\mu_{\tilde{a}}(x)$  is less than or equal to  $\alpha$  where  $\alpha \in [0, 1]$ , Fig 1(e)(f).

$${}_{\alpha}\Gamma_{\tilde{a}} = \{x \in X \mid \mu_{\tilde{a}}(x) \leq \alpha, \alpha \in [0, 1]\} \tag{4}$$

**B. Genetic Algorithms**

*Definition 5:* A GA is a searching algorithm that uses probabilistic techniques in order to find solutions to optimization problems. A GA finds the optimal solution simulating processes of natural selection. In a GA we assume that the optimization task is given by (Matoušek, 2002):

$$\begin{aligned} \min \{f(\alpha) \mid \alpha \in \{0,1\}^l\}, \\ \max \{F(\alpha) \mid \alpha \in \{0,1\}^l\}, \end{aligned} \tag{5}$$

where  $F$  is the *objective function* denoted as the *fitness function* that satisfies  $F(\alpha) > 0, \forall \alpha$ , and the minimization optimization task  $f(\alpha)$  is the objective function.  $\alpha$  is the binary string with  $l$  length. According to these concepts, we code a set of potential solutions called chromosomes (a binary string of genes) where the quality of the solution is given by their fitness function. A set of chromosomes is called population (Sharbafi, Herfeh, Shakiba, & Nejad 2006). A GA randomly generates an initial population. For the next generation, the GA haphazardly takes a set of chromosomes according to a probability in relation to their fitness, creating new generations (offspring), to modify the offspring we use genetic operators (selection, crossover and mutation), and this process is iterative until the stop criteria has been satisfied or the limit of the generations is reached (Srisutapan, Kijirikul, 2012).

**Selection:** It is the process in charge of fitness function for selecting chromosomes to reproduce. This selection is given by a probability function (Krevice, 2012):

$$P_s(x_i) = \frac{f(x_i)}{\sum_{k=1}^n f(x_k)} \tag{6}$$

where  $f(x_i)$  is the fitness function of the population vector:

$$X = \{x_1, x_2, \dots, x_n\} \tag{7}$$

**Crossover:** It is the process of exchanging genes between two individuals that are reproducing (a binary string that code an element of the genotype function).

**Mutation:** It is the process to alter chromosomes where Krevice (2012) suggest a probability to change  $i$ .  $i$  vary among 1 to  $n$ . For each  $i$  a random number is selected between 1 and 0. If the number is less than the probability then the bit is changed.

The application of a GA to solve a problem requires to determine:

- A genetic representation (*genotype*) of the solutions of the problems (*phenotype*)
- Randomly create a set of initial solutions population
- An evaluation function (fitness) which provides a quality values to each chromosome
- Operators which modify the genetic composition of the descendants during reproduction
- Values for the parameters used as population size, probabilities to apply the genetic operators, etc. (Herrera, 2005).

### C. Definition of Fuzzy Sets Optimized with Genetic Algorithms

Many problems have been solved with GAs and FL as electromagnetic field (Damousis, Satsios, Labridis & Dokopoulos, 2002) or optimization problem (Gallova, 2009). GA is a thecnique very popular of *Evolutionary Algorithms* (EA). *Genetic Fuzzy Systems* and *Fuzzy Evolutionary Algorithms* are two approaches to integrate FL and EA. In the first one, EAs are employed for solving optimization and problems retated with fuzzy systems. The second ones use fuzzy tools and fuzzy logic-based techniques to improve performance through modeling EAs' components for adapting evolutionary algorithm control parameter, (Herrera, Lozano, Mumford & Jain, 2009). In the last one, fuzzy sets are intrinsically involved in GA. In this study, we considered the first approach to optimize the function that describes a phenomenon like growing population, academic achievement, etc., according to linguistic variables.

*Definition 6:* The algorithm to find fuzzy sets that are optimized the solution of an objective function using GAs is defined as *Fuzzy Sets Optimized with Genetic Algorithm (FSOGA)*.

By the last definition we inferred that a GA is not modified and we must pay attention on chromosomes definition of fuzzy sets and the objective function. The algorithm is showed on the next paragraph.

*Algorithm 7:* Procedure FSOGA

read data  $P_{\alpha}$  // read studied-population-data

defined GA parameters // generations-number (G),

population-size (n), mute probability ( $m_p$ )

initialize population  $P(0)$  // defined of parameters of fuzzy sets according to population-size and limits of fuzzy sets over the definition on antecedent and consequent independent variables

for  $t = 1$  to G // begin the evolutions according to limit of generation-number

random:  $P(t - 1)'$  // population's subjects randomly ordered

crossover:  $P(t - 1)'' \leftarrow P(t - 1)' \cup \text{crossover}(P(t - 1)')$

// now, population size is double

mutate:  $P(t - 1)''' \leftarrow P(t - 1)'' \cup \text{mutate}(P(t - 1)'')$

// now, population size is quad

FS construction // fuzzy sets construction with  $P(t - 1)'''$

evaluate // evaluate fitness function using FSs of each

$M \in P$  to describe  $P_{\alpha}$

selecting  $n$  best subjects of  $P(t - 1)'''$

end (for)

end (FSOGA)

#### D. Academic Approach Estimation

The AA *estimation* is one of the most critical tasks in the students’ attention management, because it is impossible to deal every case in order to improve the terminal efficiency estimation. So it is important to build a model which can be able to predict the AA *estimation*. The techniques for *estimation* can be classified into: Expert Judgment, Algorithmic Models, and Techniques based on Soft-Computing (López-Martín, 2011).

With the proposed algorithm, FSOGA, we obtain a model of a monotonic crescent curve, according to the independent variable, *aptitude-test*, and *first-assessment* that is the dependent variable. *aptitude-test* data are obtained with the college admission test students and *first-assessment* are accomplish with the student grades obtained after the first semester. The *first-assessment* described AA after first semester.

The evaluating criterion for AA *estimation* in this study is the *Magnitude of Error Relative* (MER), which is defined by (Kitchenham, MacDonell, Pickard & Shepperd, 2001):

$$MER = \frac{|Actual\_AA - Predicted\_AA|}{Predicted\_AA} \tag{8}$$

Where the *Expert judgment* of the developer is the *Predicted* AA and the *Real* AA is the *Actual* AA. The MER value is calculated for each observation *i* whose AA is predicted. The aggregation of MER over multiple observations (N) can be achieved through the mean (MMER) as follows:

$$MMER = \frac{1}{N} \sum_{i=1}^N MER_i \tag{9}$$

The accuracy of an estimation technique is inversely proportional to the MMER.

### III. FSGA Implementation

Each fuzzy set  $\tilde{a}$  represent a linguistic variable and has triangular form defined with three parameters i.e.

$$\tilde{a} = (w_1, w_2, w_3) \tag{10}$$



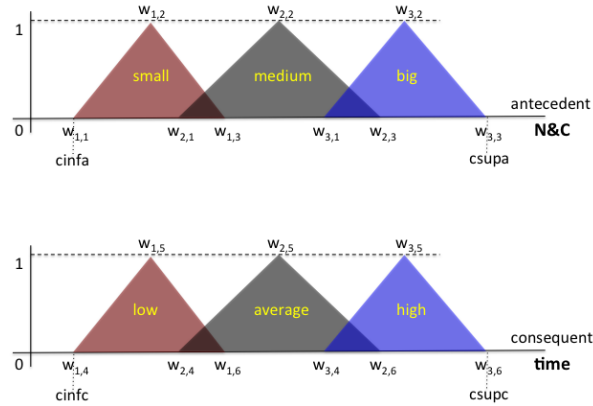


Fig. 2. A member of population defined with FSs.

We used the relation between *aptitude-test* and *first-assessment* (AA) and they are the antecedent and the consequent, respectively, in a relation between them. With this, three rules are made:

- if *small* then *low*
- if *medium* then *average*
- if *big* then *high*

The rules are simple because they have only one antecedent and one consequent, respectively. Now, we say that M is a member (*subject*) of the population P,  $M \in P$ . Each member M has FSs which are involved in all rules and is defined as:

$$M = (w_{r,1}, w_{r,2}, w_{r,3}, w_{r,4}, w_{r,5}, w_{r,6}) \mid r = 1 \dots \varphi \tag{11}$$

where  $\varphi$  is the number of rules. Obviously, each M is a *candidate solution*. Because of the three rules, when  $r = 1$  we have that  $w_{1,1}, w_{1,2}, w_{1,3}$  describe the small FS and  $w_{1,4}, w_{1,5}, w_{1,6}$  describe the low FS. When  $r = 2$ ,  $w_{2,1}, w_{2,2}, w_{2,3}$  describe the medium FS and so on, Fig. 2. M has the following matrix representation:

$$M = \begin{bmatrix} w_{1,1} & w_{1,2} & w_{1,3} & w_{1,4} & w_{1,5} & w_{1,6} \\ w_{2,1} & w_{2,2} & w_{2,3} & w_{2,4} & w_{2,5} & w_{2,6} \\ w_{3,1} & w_{3,2} & w_{3,3} & w_{3,4} & w_{3,5} & w_{3,6} \end{bmatrix} \tag{12}$$

$$\equiv \begin{bmatrix} \tilde{a}_{small} & \tilde{a}_{low} \\ \tilde{a}_{medium} & \tilde{a}_{average} \\ \tilde{a}_{big} & \tilde{a}_{high} \end{bmatrix} \tag{13}$$

M can be represented as two blocks of fuzzy sets. In the first, antecedent fuzzy sets are included. The second has consequent fuzzy sets, i.e.  $M \equiv [\tilde{a}_A, \tilde{a}_C]$ . This representation is used to compute crosses.

Then the genetic representation (genotype, (G)) has the form:

$$G = \begin{bmatrix} g_{1,1} & g_{1,2} & g_{1,3} & g_{1,4} & g_{1,5} & g_{1,6} \\ g_{2,1} & g_{2,2} & g_{2,3} & g_{2,4} & g_{2,5} & g_{2,6} \\ g_{3,1} & g_{3,2} & g_{3,3} & g_{3,4} & g_{3,5} & g_{3,6} \end{bmatrix} \tag{14}$$

where  $g_{i,j} \mid i = 1 \dots 3, j = 1 \dots 6$  is a specific gene that has a binary value. For the *small* linguistic variable, its genotype is the vector  $g_{small} = [g_{1,1} \ g_{1,2} \ g_{1,3}]$ , then all linguistic variables' *genotype* are distributed as follow:

$$G \equiv \begin{bmatrix} g_{small} & g_{low} \\ g_{medium} & g_{average} \\ g_{big} & g_{high} \end{bmatrix} \tag{15}$$

G can be represented by antecedent and consequent genotype i.e.  $G \equiv [g_A, g_C]$ . This *genotype* representation is used in FSGA to compute the mutation. The population is defined as:

$$P = Mk \mid k = 1 \dots n \tag{16}$$

$$= (w_{\varphi-i+j,1}, w_{\varphi-i+j,2}, w_{\varphi-i+j,3}, w_{\varphi-i+j,4}, w_{\varphi-i+j,5}, w_{\varphi-i+j,6})$$

$$\mid i = 0 \dots n - 1, j = 1 \dots \varphi$$

where  $n$  is the *population size*. Because of three rules, the population is a matrix with the following form:

$$P = \begin{bmatrix} w_{3-i+1,1} & w_{3-i+1,2} & w_{3-i+1,3} & w_{3-i+1,4} & w_{3-i+1,5} & w_{3-i+1,6} \\ w_{3-i+2,1} & w_{3-i+2,2} & w_{3-i+2,3} & w_{3-i+2,4} & w_{3-i+2,5} & w_{3-i+2,6} \\ w_{3-i+3,1} & w_{3-i+3,2} & w_{3-i+3,3} & w_{3-i+3,4} & w_{3-i+3,5} & w_{3-i+3,6} \end{bmatrix} \quad (17)$$

The parameters of each fuzzy sets are on a specific *search space* and these spaces are bounded with constraints. Constraints are convenient because the convergence could be fast, but this is not strictly necessary for other problems. Antecedent fuzzy sets are formed with the follow constraints:

$$w_{3-i+1,1} = \text{cinf}a \quad (18)$$

$$w_{3-i+3,3} = \text{csup}a$$

$$w_{3-i+1,1} \leq w_{3-i+1,3} \leq \frac{3}{4}[w_{3-i+3,3} - w_{3-i+1,1}]$$

$$w_{3-i+1,1} \leq w_{3-i+1,2} \leq w_{3-i+1,3}$$

$$\frac{1}{4}[w_{3-i+3,3} - w_{3-i+1,1}] + w_{3-i+1,1} \leq w_{3-i+3,1} \leq w_{3-i+3,3}$$

$$w_{3-i+3,1} \leq w_{3-i+3,2} \leq w_{3-i+3,3}$$

$$w_{3-i+1,1} \leq w_{3-i+2,1} \leq w_{3-i+1,3}$$

$$w_{3-i+3,1} \leq w_{3-i+2,3} \leq w_{3-i+3,3}$$

$$w_{3-i+2,1} \leq w_{3-i+2,2} \leq w_{3-i+2,3}$$

The constraints to form consequent fuzzy sets are:

$$w_{3-i+1,4} = \text{cinf}c \quad (19)$$

$$w_{3-i+3,6} = \text{csup}c$$

$$w_{3-i+1,4} \leq w_{3-i+1,6} \leq \frac{3}{4}[w_{3-i+3,6} - w_{3-i+1,4}]$$

$$w_{3-i+1,4} \leq w_{3-i+1,5} \leq w_{3-i+1,6}$$

$$\frac{1}{4}[w_{3-i+3,6} - w_{3-i+1,4}] + w_{3-i+1,4} \leq w_{3-i+3,4} \leq w_{3-i+3,6}$$

$$\begin{aligned}
 w_{3-i+3,4} &\leq w_{3-i+3,5} \leq w_{3-i+3,6} \\
 w_{3-i+1,4} &\leq w_{3-i+2,4} \leq w_{3-i+1,6} \\
 w_{3-i+3,4} &\leq w_{3-i+2,6} \leq w_{3-i+3,6} \\
 w_{3-i+2,4} &\leq w_{3-i+2,5} \leq w_{3-i+2,6}
 \end{aligned}$$

In FSGA, the initial population  $P(0)$  is randomly ordered  $P'(0)$ . This will ensure a cross of features among population's members.

$$P'(0) = \begin{bmatrix} M'_1 \\ \vdots \\ M'_n \end{bmatrix} \equiv \begin{bmatrix} \tilde{\alpha}'_{A_1} & \tilde{\alpha}'_{C_1} \\ \vdots & \vdots \\ \tilde{\alpha}'_{A_n} & \tilde{\alpha}'_{C_n} \end{bmatrix} \tag{20}$$

In the *crossover* step a new population of sons  $M^c$  is generated. Two parents will generate two sons. The sons have genes crossed of the parents as:

$$M^c = \begin{bmatrix} \tilde{\alpha}'_{A_1} & \tilde{\alpha}'_{C_2} \\ \tilde{\alpha}'_{A_2} & \tilde{\alpha}'_{C_1} \\ \vdots & \vdots \\ \tilde{\alpha}'_{A_{k-1}} & \tilde{\alpha}'_{C_k} \\ \tilde{\alpha}'_{A_k} & \tilde{\alpha}'_{C_{k-1}} \end{bmatrix} \tag{21}$$

$M^c$  will be added to parent population  $P'(0)$  and it generates  $P''(0)$ .

$$P''(0) = \begin{bmatrix} P'(0) \\ M_1^c \\ \vdots \\ M_k^c \end{bmatrix} \equiv \begin{bmatrix} M''_1 \\ \vdots \\ M''_m \end{bmatrix} \mid m = m + k \tag{22}$$

$$k = n \text{ if } n \text{ is even number}$$

$$k = n - 1 \text{ if } n \text{ is uneven number}$$

The mutate step generates a new population of sons  $M^d$  that will be added to  $P''(0)$

$$P'''(0) = \begin{bmatrix} P''(0) \\ M_1^d \\ \vdots \\ M_m^d \end{bmatrix} \equiv \begin{bmatrix} M_1^{''''} \\ \vdots \\ M_q^{''''} \end{bmatrix} \mid q = 2m \tag{23}$$

The mutation is operated over  $g_A$  and  $g_C \in G$ . In this step each  $M \in P''(0)$  could be mutated in  $\tilde{a}_A''$  and/or  $\tilde{a}_C''$ . Finally,  $n$  best subjects are selected from  $P'''(0)$  and they form a new generation  $P(1)$ .  $P(0)$ 's process will be reproduced for  $P(1)$ .

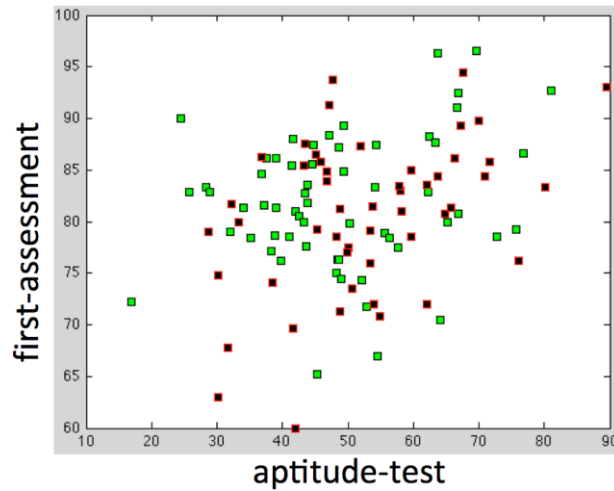


Fig. 3. Population.

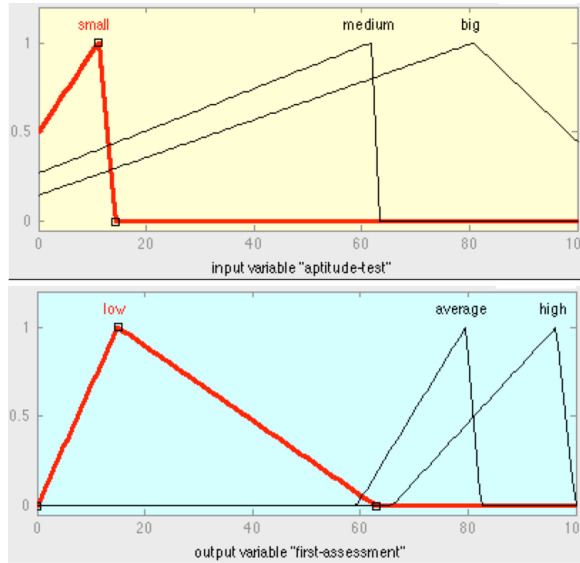


Fig. 4. Fuzzy Sets that are a model solution.

**IV. Experiment and Result**

The problem to address is to find the best representation to obtain effort estimation. Study population ( $P_d$ ) is obtained with (González-Castolo, Ramos-Cabral & Hernández-Gallardo, 2014) data and this is showed in Fig. 3. Clear and black dots are information to create and validate the model respectively.

FSGA obtained a solution using 1000 cycles (generations) that is showed as:

$$\tilde{S} = \begin{bmatrix} -11.09 & 11.1 & 14.33 & 0 & 14.98 & 62.88 \\ -22.83 & 61.71 & 63.31 & 59.22 & 79.5 & 82.47 \\ -13.85 & 80.84 & 115.28 & 65.81 & 96.29 & 100.00 \end{bmatrix}$$

Here the *small* variable linguistic is represented as  $\tilde{a}_{small} = (-11.09, 11.1, 14.33)$ , the *medium* variable linguistic is  $\tilde{a}_{medium} = (-22.83, 61.71, 63.31)$  and so on. The solution is corresponded with fuzzy sets showed in Fig 4.

The defuzzyfication is processed with centroid method. The Fig. 5 show  $\tilde{S}'_s$  defuzzyfication. For example if *aptitude-test* = 50 of score then *first-assessment* = 81.9 rate.

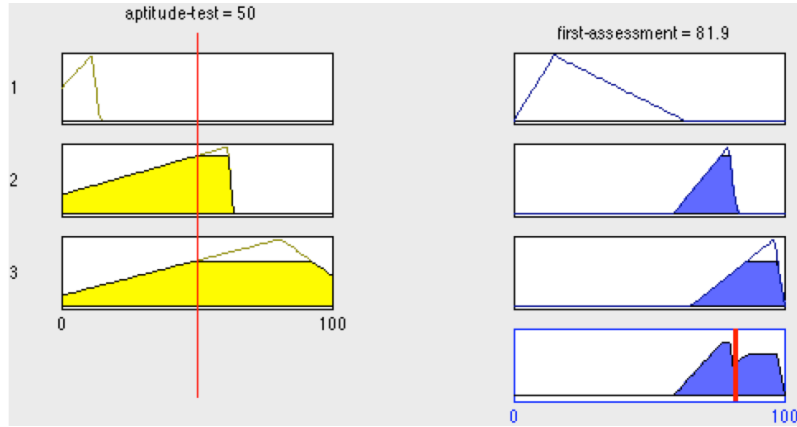


Fig. 5. Defuzzification with centroid method.

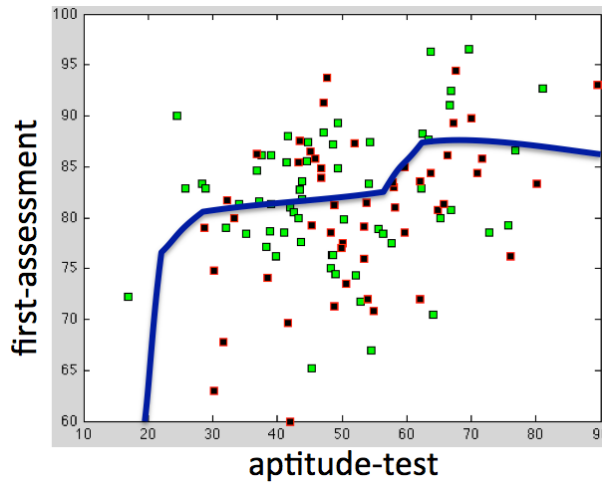


Fig. 6. Representation of the model obtained.

The model obtained is showed in Fig. 6. The fist (0, 20] and last [80, 90) trajectory is not important because the  $P_d$  doesn't have enough information here.

The MMER obtained is shown to next:

$$MMER = 0.0579$$

MMER is improved with respect to linear regression solution.  $MMER_{vali}$  is the result of MMER using the information to validate the model and obtained the following value:

$$\text{MMER}_{\text{vali}} = 0.0671$$

## Conclusion

This paper addressed the optimization procedure to AA model that is traditionally described using linear regression. We proposed FSOGA algorithm that is a genetic algorithm that find FSs that minimize the objective function MMER. In this point, MMER is improved with respect other results but the main contribution is the procedure to find FS using a Genetic Algorithm to model an specific problem. This process could be applied to solve many problems as effort to modify or create computers programs. The future research is using multi input variables to AA estimate.

## References

- Damousis, I.G., Satsios, K.J., Labridis, D.P., & Dokopoulos, P.S. (August (2002)). Combined Fuzzy Logic and Genetic Algorithm Techniques Application to an Electromagnetic Field Problem. Fuzzy Sets and Systems-Control and applicatios archive, 129(3), 371-386.
- Durán, E. B., & Costaguta, R. N. (2007). Minería de datos para descubrir estilos de aprendizaje. Revista Iberoamericana de Educación, 42(2), 6.
- Gallova, S. (July 1 - 3 (2009)). Genetic Algorithm as a Tool of Fuzzy Parameters and Cutting Forces. Optimization Proceedings of the World Congress on Engineering 2009 , 1.



- González-Castolo, J. C., & López-Mellado, E. (September (2007)). Approximating State of DES Using Fuzzy Timed Petri Nets. IEEE International Conference on Automation Science and Engineering (CASE 2007), 722-728.
- Gonzalez-Castolo, J.C., Ramos-Cabral, S., & Hernández-Gallardo, S. C. (Enero (2014)). Construcción de un modelo difuso optimizado del aprovechamiento académico de estudiantes en educación superior. 3er Congreso Virtual sobre Tecnología, Educación y Sociedad. ISSN 2007-7475, 1-14.
- Herrera, F., Lozano M., Mumford, C.L., & Jain L.C. (2009). Fuzzy Evolutionary Algorithms and Genetic Fuzzy Systems: A Positive Collaboration between Evolutionary Algorithms and Fuzzy Systems. Computational Intelligence Springer-Verlag Berlin Heidelberg, 1, 83-130.
- Herrera, F. (2005). Genetic Fuzzy Systems: Status, Critical Considerations and Future Directions. International Journal of Computational Intelligence Research.
- Kitchenham, B.A., MacDonell, S.G., Pickard, L.M., & Shepperd, M.J. (2001). What Accuracy Statistics Really Measure. IEE Proceedings Software.
- Klir, G. J., & Yuan, B. (1995). Fuzzy Sets and Fuzzy Logic, Theory and Applications. NJ: Prentice Hall.
- Krevice, P. E. (Retrieved May 12 (2012)). The Genetic Algorithm in Computer Science. . MIT Undergraduate Journal of Mathematics.
- Larrañaga, M., Elorriaga, J. A., Rueda, U., & Arruarte, A. (2004). Modelado de grupos en actividades de aprendizaje basado en mapas conceptuales. Inteligencia artificial. Revista Iberoamericana de Inteligencia Artificial, 8(24), 131-148.

- López-Martín, C. (2011). A Fuzzy Logic Model for Predicting the Development Effort of Short Scale Programs Based Upon Two Independent Variables. *Applied Soft Computing*, 11, 724-732.
- Lye, C. T., Ng, L. N., Hassan, M. D., Goh, W. W., Law, C. Y., & Ismail N. (2010). Predicting Pre-university Student's Mathematics Achievement. *Procedia-Social and Behavioral Sciences*, 8, 299-306.
- Matoušek, R. (2002). Realisation of Fuzzy-Adaptive Genetic Algorithms In a Matlab Environment. Institute of Automation and Computer Science, Brno University of Technology.
- Mendel, J. M. (1995). Fuzzy Logic Systems for Engineering: A Tutorial. *IEEE*, 83.
- Renders, J. M., & Flasse, S.P. (April (1996)). Hybrid Methods Using Genetic Algorithms for Global Optimization. *IEEE Transactions on Systems, Man, and Cybernetics-Part b: Cybernetics*, Vol. 26, No 2.
- Rowcliffe, P., & Feng, J., (September 2008). Training Spiking Neuronal Networks With Applications in Engineering Tasks. *IEEE Transactions on Neural Networks*, 19 (9).
- Shannon, R. E., & Bernal, F. A. (1988). Simulación de sistemas: Diseño, Desarrollo e implantación. México : Trillas.
- Sharbafi, M. A., Herfeh, M., Shakiba, L. C., & Nejad, A. M. (2006). An Innovative Fuzzy Decision Making Based Genetic Algorithm. *World Academy of Science, Engineering and Technology*, 19.
- Srisutapan, J., & Kijirikul, B. (Retrieved May 19 (2012)). Fuzzy Logic And Genetic Algorithm For Optimizing. The Approximate Match Of Rules Based On Back propagation Neural Networks.

- Tang, K.S., Man, K.F., Kwongs, S., & He, Q. (1996). Genetic Algorithms and Their Applications. IEEE Signal Processing Magazine.
- Tarifa, E. (2001). Teoría de Modelos y Simulación. Facultad de Ingeniería: Universidad de Jujuy.
- Varnamkhasti, M. J., Lee, L. S., Abu Bakar, M. R., & Leong W. J. (2012). A Genetic Algorithm with Fuzzy Crossover Operator and Probability. Hindawi Publishing Corporation Advances in Operations Research, Article ID 956498.
- Wang, L., & Langari, R. (February (1996)). Complex Systems Modeling via Fuzzy Logic. IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics, 26 (1).
- Warren, S. (1994). Neural Networks and Statistical Models.. Proceedings of the Nineteenth Annual SAS Users Group International Conference
- Zadeh, L. A.. (1965). Fuzzy Sets. Information and Control , 8, 338-353.
- Zhi, L., & Han-Xiong, L. (December (2005)). A Probabilistic Fuzzy Logic System for Modeling and Control. IEEE Transactions on Fuzzy Systems, 13 (16).